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U.S. Department of Commerce, Bureau of Standards

RESEARCH PAPER RP653

Part of Bureau of Standards Journal of Research, vol. 12, March 1934

STATISTICAL INVESTIGATION OF THE UNIFORMITY OF GRADES OF 1,000 LOVIBOND RED GLASSES

By Geraldine K. Walker

ABSTRACT

The need for uniformly graded Lovibond red glasses has long been urgent among those engaged in the vegetable- and cottonseed-oil trade. Huge quantities of oil are yearly bought and sold with color as an important contributing factor in the rules governing the transactions. The grade of the oil depends upon its color and the color is defined in terms of Lovibond glasses. The calibration of these red glasses in terms of the standard unit and scale, known as the Priest-Gibson unit and scale, has been in progress at the Bureau of Standards since 1928. By direct comparison with the working standards a new value or regrade numeral is assigned to each glass submitted for test. The new value is the effective additive value when the glass is used in combination with a 35-yellow (35Y) glass. The direct-comparison method of calibration and the apparatus are described.

The statistical investigation of the data resulting from the calibration of the first 1,000 red glasses shows that variations of a whole unit frequently exist among glasses of similar Lovibond numeral. The several illustrations display in different ways the discrepancies found among these glasses. For purposes of the oil trade, not only should all glasses of the same color bear the same numeral, but, also, those of different colors should bear the corresponding proper numerals according to a scale which is accurately additive. Both regular and erratic departures from additivity have been demonstrated in the Lovibond scale as embodied in these 1,000 red glasses combined with 35Y, and the additive nature of the Priest-Gibson scale has been confirmed.

CONTENTS

I. INTRODUCTION

The standardization of Lovibond red glasses [1] was undertaken at the request [2] of the Society of Cotton Products Analysts, now Ameri-

[1] A description of the Lovibond color scale is given on pages 6 and 7 of "Colour Measurement", a recently published brochure of The Lovibond Tintometer, The British Drug Houses Pattern, Pat. No. 299194. For the method of construction of these scales of red, yellow, and blue glasses, see J. W. Lovibond, Light and Colour Theories, E. & F. N. Spon, London, pp. 29–30.

[2] As early as 1912 an appeal was made by the Society of Cotton Products Analysts to the Bureau of Standards to test Lovibond tintometer glasses (Report of Proceedings of the Fourth Annual Meeting of the Society of Cotton Products Analysts, Chicago, Ill., June 21, 1913). Regarding the work on the standardization of Lovibond glasses prior to 1927, see the prefatory statement by I. G. Priest in a paper by K. S. Gibson and F. K. Harris, The Lovibond Color System—A Spectrophotometric Analysis of the Lovibond Glasses, B.S. Scientific Paper 547, February 1927.

can Oil Chemists' Society, whose members are engaged in the vegetable oil industry. The oil chemists use these red glasses in combination with Lovibond yellow glasses to assign grades to their oil products, the grade and price depending upon the color. Discrepancies [3] were occurring in the grades assigned to the same samples of oil in the laboratories of producer and consumer, where different sets of glasses were in use. Notable inconsistencies between the colors of the red glasses and their Lovibond numerals [4], N, were known to exist and it was desired to eliminate as far as possible any differences in the grades of oils due to these inconsistencies. The grade of an oil is defined by two numbers which represent the combination of red and yellow glasses similar in color to the oil. One is the sum of the Lovibond numerals on the red glasses and the other is the Lovibond numeral on the yellow glass used in the combination. In decisions of greatest importance, the red glasses are used in combination with Lovibond 35 yellow (35Y) [5] and it has already been shown [6] that the lack of uniformity among 35Y glasses is relatively unimportant.

Uniformity among the red glasses, however, is an important factor. All glasses of the same color should bear the same numeral, and in addition, since they are usually used in combinations of two or three, those of different color should bear the corresponding proper numerals according to a scale which is accuratively additive. That is, when two or more glasses are used in combination, they should be equivalent [7] to a single glass of numeral equal to the sum of those in the combination. This is called the additivity condition, and by its application a relatively small number of glasses by various combinations may be made to yield the many standards of color needed in grading vegetable oils.

When combined with 35Y, the red glasses failed to meet these conditions to the extent described in the present paper. Furthermore, there was no satisfactory additive scale or set of standards available by means of which the red glasses could be calibrated. The standardization of red glasses has, therefore, consisted of two principal parts:

1. *The standardization of the Lovibond red glasses of the Bureau's own set, designated as B.S. 9940, in combination with Lovibond 35Y.*— The Society of Cotton Products Analysts and the Interstate Cotton Seed Crushers' Association adopted this set as "standard" as early as 1913. The derivation of this required scale, by means of which new numerals, N'', satisfying the additivity condition, were assigned to glasses of set B.S. 9940, is described in detail in a paper now in prep-

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[3] Some of the possible sources of the discrepancies mentioned in B.S. Scientific Paper 547, p. 10, have been studied and results presented in the following publications:

D. B. Judd and G. K. Walker, A Study of 129 Lovibond Red Glasses with Respect to the Reliability of their Nominal Grades, Oil and Fat Industries, 5, pp. 16–26, January 1928.

I. G. Priest, Tests of Color Sense of A.O.C.S. Members and Data on Sensibility to Change in Lovibond Red, Oil and Fat Industries, 5, pp. 63–73, March 1928.

D. B. Judd, Effect of Temperature Change on the Color of Red and Yellow Lovibond Glasses, B.S. Jour. Research, vol. 1, (RP31), pp. 859–866, November 1928.

I. G. Priest, D. B. Judd, K. S. Gibson, and G. K. Walker, Calibration of Sixty-five 35-Yellow Lovibond Glasses, B.S.Jour. Research, vol. 2 (RP58), pp. 793–819, April 1929.

[4] The numeral engraved on each glass, presumably by the Lovibond establishment, will be represented by N and will be referred to in the present paper by use of any one of the terms: Lovibond numeral, nominal grade, or maker's numeral. This Lovibond numeral is nominally a means of differentiating between the colors of the various members of the red scale, which is embodied in 155 red glasses ranging in nominal grade from .01 to 20.0 Lovibond units. A brief but comprehensive description of the Lovibond system has been published in B.S. Scientific Paper 547, p. 11. Similar numerals are used in the Lovibond yellow and blue scales.

[5] Rules governing transactions between members of the National Cottonseed Products Association, Inc. successors to the Interstate Cotton Seed Crushers' Association, Inc. See article 4, rules 60–65.

[6] B.S. Research Paper 58, p. 819.

[7] Their equivalence is considered established if they evoke the same chromaticity under like conditions; see footnote 16, p. 274.

aration.[8] These glasses having N'' values will be referred to as standard glasses.

2. The standardization (regrading) of red glasses submitted to the Bureau for calibration in terms of the standard N'' scale.—This second standardization is the subject of the present paper. The routine method of calibration now used was adopted in the fall of 1927 and the routine testing of red glasses submitted for calibration has been in progress since January 1928. From September 1927 to July 1928 this work was performed in cooperation[9] with the American Oil Chemists' Society. Since that time, the Bureau of Standards has continued the routine grading and in March 1929 the glasses calibrated totaled 1,000.

The present paper describes the apparatus and the procedure used in the calibration of these 1,000 glasses and gives the results of a statistical analysis of their regrade numerals relative to their respective nominal grades.

It may be emphasized that all values of N'' given in this paper refer to the red glasses when combined with 35Y. It is doubtless true that the regrade numerals so obtained for the red glasses would also hold for a considerable range of yellow glasses on both sides of 35Y, but these numerals would not necessarily be valid for the red glasses used alone or with a Lovibond yellow glass of small numeral.

II. STANDARD GLASSES

The method adopted for the routine grading of red glasses is known as a direct-comparison method because a submitted glass is compared directly with a standard glass or combination of standard glasses. For convenience in grading, it was desirable to have as standards a series of glasses ranging in Priest-Gibson numerals, N'', from 0.1 to 1.0 in multiples of 0.1 and from 1.0 to 10.0 in multiples of 1.0. The selection of certain glasses for standards was, therefore, largely governed by the degree of approximation of their N'' values to these desired multiples. In table 1 are listed the values of N and N'' for those standards of B.S. 9940 which were most extensively used in the calibration of the 1,000 glasses. As already noted, Lovibond 35Y was used in deriving the N'' values of these standards.

TABLE 1.—*Standards of B.S. 9940 used in routine grading of Lovibond red glasses*

N[1]	N''[2]	N	N''
0.02	0.104	2.8	2.99
.13	.199	3.9	3.94
.18	.296	4.9	5.08
.28	.365	6.8	6.85
.38	.495	7.6	7.59
.46	.593	9.0	9.17
.56	.711	9.8	10.00
.64	.806	12.0	12.17
.74	.900	13.0	12.98
.86	1.003	17.0	17.12
1.8	1.95		

[1] N=Maker's numeral.
[2] N''=Regrade numeral in Priest-Gibson units. These values are uncertain in the second decimal place. Nevertheless, they have been used, as given, in the calibration of tested glasses. Lovibond 35Y was used in deriving the N'' values of these standards.

[8] The unit and scale were established by I. G. Priest and K. S. Gibson in May 1927, and a brief description was given orally both at the Convention of the A.O.C.S. held in Memphis, May 1927, and at the O.S.A. held in Schenectady, N.Y., October 1927. For abstract, see Jour. Opt. Soc. Am., and Rev. Sci. Inst., 16, p. 116, February 1928. The symbol N'' was used because the symbol N' had already been used in the derivation.

[9] The author was at that time employed at the Bureau of Standards as a Research Associate representing the A.O.C.S.

III. TESTED GLASSES

The 1,000 glasses, to which regrade numerals, N'', have been assigned range in nominal grades, N, from 0.09 to 20.0 Lovibond units. All engravings on each glass as received at the Bureau have been recorded, because of interest and possible service in establishing the origin, age, and history of the individual glass. Facsimiles of the various types of engravings are on file. For the same reasons, the thickness [10] of the center in millimeters, the character of the edges, and the surface imperfections in the middle third of the area have been carefully noted and are on record. For precision work, it would be desirable to have standards whose surfaces are plane and parallel, and which are free from pits, scratches, striae, bubbles, and haze. Very few of the glasses tested could be considered as ideal standards in these respects, yet none have been recommended for rejection. Some of the defects noted are in all probability the result of usage.

During the period when regrading was being done in cooperation with the A.O.C.S., the glasses were submitted with "AOCS numbers" engraved by A. W. Putland, who kept the key to these numbers, this being the only means of identification. Since then, Bureau of Standards test and serial numbers have served as a means of identification.

IV. APPARATUS

The apparatus (fig. 1) consists essentially of a Martens photometer [11] with auxiliary parts providing both a support for the glasses and a source of illumination.[12] The main parts are: Lamp inclosure, Davis-Gibson liquid filter, biprism and lens system, black chamber, Martens photometer, and eyeshield.

The field of view in the photometer is a 6-degree circular field divided along its vertical diameter. Each half of the circular field is illuminated by light diffusely reflected from an inclosure lined with magnesium oxide on white porcelain enamel and containing two 100-watt gas-filled tungsten lamps. The light reflected from the center of the rear surface of the inclosure passes through a window opposite the rear surface. The color temperature of the reflected light is maintained approximately at 2,848 K by means of a rheostat and voltmeter.

Before entering the photometer, the light passes through a Davis-Gibson filter [13] which gives it a spectral energy distribution approximating that of noon sunlight (Abbot-Priest sun). The filter consists of two solutions [14] encased in a double glass cell, the principal constituent being a blue solution which approximately converts the yellowish incandescent light to sunlight. By means of lenses and a biprism the artificial sunlight is directed through the black chamber into the photometer to give a uniform photometric field. The black chamber

[10] There is no correlation between thickness and grade, because the glasses are flashed. The average size of a Lovibond glass is approximately 50 by 17 by 2.5 mm.

[11] For original technical description, see Phys. Zeitschrift, 1, pp. 299–303, 1900.

[12] The original source of illumination was that of natural north skylight transmitted through milk glass. However, the grading of glasses was greatly delayed because the illumination was frequently insufficient. The present equipment, using artificial sunlight, provides a constant illumination and furthermore yields a much higher brightness than that previously obtained. A few glasses graded with the skylight illumination were regraded with the new equipment with entirely consistent results.

[13] R. Davis and K. S. Gibson, Filters for the Reproduction of Sunlight and Daylight and the Determination of Color Temperature, B.S.Misc.Pub. 114, January 21, 1931, p. 157.

[14] The filter and lamps were in use during the period of a year, from April 1928 to April 1929, the lamps being burned for about 700 hours. At the end of this time both the spectral transmission of the filter and the color temperature of the light source were determined. Results indicated that the lamps had become slightly yellower and the filter slightly bluer, so that, for our purpose, the total effect was negligible. It should be noted that little change in the grade assigned to a glass is to be expected, even with a considerable change in the relative energy distribution of the illuminant, because the glass tested and the combination of standards matching it had nearly the same spectral transmissions.

contains three slots, each slot of sufficient size to hold two glasses, one in each beam. The central portion of each glass, when in position in the black chamber, is effective in transmitting light to the photometer field.

By a rotation of the dial attached to the nicol prism, the two halves of the photometric field can be made equal in brightness. Attached

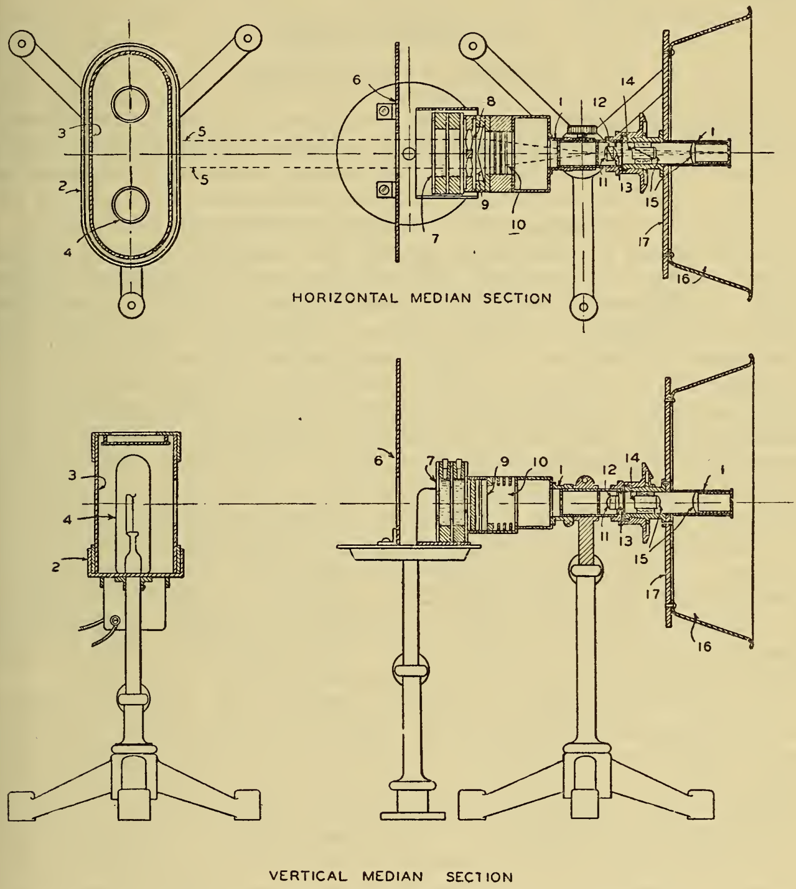

FIGURE 1.—*Apparatus used in the routine grading of Lovibond red glasses.*

In the vertical median section the supports are shown in outline rather than in section.

1. Martens photometer
2. Lamp inclosure
3. Magnesium oxide on white enamel
4. 100-watt gas-filled tungsten lamp
5. Rays from light source
6. Light shield
7. Davis-Gibson liquid filter
8. Lens
9. Biprism
10. Black chamber for holding Lovibond glasses
11. Objective lens of Martens photometer
12. Wollaston prism
13. Biprism of Martens photometer
14. Analyzing nicol prism
15. Ramsden ocular
16. Eyeshield
17. Knurled edge

to this dial is a pan-shaped eyeshield, painted black, with a knurled edge, the circumference of which is about 63 cm. This eyeshield serves two purposes: (1) The unused eye may remain open during observations; (2) due to the large lever arm of the knurled edge, fine adjustments of the nicol prism are possible.

A few other details of the apparatus may also be mentioned. A metal holder (not shown in fig. 1) for the 35Y glass fits over the

ocular of the Martens photometer and is attached by set screws to the tube containing the lens for bringing the divided circular field into focus. A metal shield between the lamp enclosure and liquid filter prevents glare from the light source when the observer interchanges glasses. The various parts of the apparatus are all mounted on the same base and adjustments for height are possible.

V. PROCEDURE

1. DIRECT COMPARISON

As stated previously, the glass to be tested is compared directly under artificial sunlight illumination with the standard glass or combination of standard glasses. The central portions [15] of each are viewed in juxtaposition in the Martens photometer through the 35Y glass placed over the ocular. A standard glass or combination of standard glasses is selected giving the best possible match in chromaticity.[16] Since the smallest intervals between the standard glasses in use are differences of approximately 0.1 of an N'' unit, estimates between the tenths are made whenever differences of this small magnitude can be detected. When a combination of two or more standard glasses is necessary to secure the match, the glass being tested is combined with a corresponding number of clear glasses, so that the light illuminating one half of the photometric field passes through as many reflecting surfaces as that illuminating the other half. Three glasses are the maximum number ever used in one half of the photometric field. By thus limiting the number of standard glasses errors due to the accumulation of small errors in the standards are prevented. Because of this limitation, and the restricted number of standard glasses available, it is frequently necessary, in order to obtain the N'' value for the glass tested to combine a standard glass with the glass being tested and subtract its N'' value from that of the standard glasses in the other half of the photometric field.

After the first decision, the glass being tested and the standards are interchanged in position and a second decision made. An average of these two readings is recorded as one determination. The final grade of each glass tested is based upon at least two separate determinations embodying two different combinations of standards. To increase the reliability of the average grade, a third or fourth combination of standard glasses is occasionally used, especially when the glass tested is of nominal grade greater than 10.0 Lovibond units.

Decisions are made near the position of brightness match. Ease of observation resulting in a very rapid and precise decision regarding a chromaticity match is attained by keeping the fixation on the brighter half of the field close to the dividing line, while the nicol prism is rotated rapidly back and forth in a small angle through the position of brightness match.[17]

[15] Lovibond numerals, N, are based on measurements made through the ends of the glasses opposite the engraved numerals.

[16] "The term *chromaticity* may be used to characterize a color qualitatively without reference to its brilliance. Chromaticity is determined by hue and saturation together, a gray being specified by the statement that it has no chromaticity." Report of the Colorimetry Committee, Optical Society of America, Jour. Opt. Soc. Am. and Rev. Sci. Inst., 6, p. 535, August 1922.

[17] Some observers prefer to make their decisions keeping their fixation on the darker half of the field while the nicol prism is rotated through the position of brightness match. However, the decisions on these 1,000 glasses were made in the former way. This factor as an aid to detection of chromaticity differences has been mentioned in a paper by D. B. Judd, Precision of Color Temperature Measurements under Various Observing Conditions; A New Color Comparator for Incandescent Lamps, B.S. Scientific Paper 252, 5, p. 1163, 1930.

2. CROSS-CHECKING

The regrade numerals, N'', obtained by the above procedure are subject to errors from a variety of sources, viz, (1) errors in N'' assigned to the standard glasses, (2) uncertainty of observation, and (3) errors of recording. To detect important errors in the values of N'' obtained for the glasses tested, each of these glasses is compared with its successive neighbor in the same group. This intercomparison or cross-check has the decided advantage that, for the most part, a single standard of small N'' value is required, with corresponding small absolute error in its value. Consistent results between regrade and cross-check data are obtained before any regrade numerals are certified, although the cross-check data do not enter into the computations of the regrade numerals.

3. CERTIFICATION AND RELIABILITY

Regrade numerals, N'', are certified to be correct within certain tolerances depending upon the nominal grades of the glasses. These tolerances are (*a*) 0.1 of an N'' unit for glasses ranging in nominal grade from the smallest value submitted up to and including 10.0; (*b*) 0.2 of an N'' unit for nominal grades greater than 10.0 and those up to and including 16.0; (*c*) 0.4 of an N'' unit for nominal grades greater than 16.0 and those up to and including 20.0. Theoretically, the tolerances should not be assigned in this step-like fashion, since previous tests have shown that the red-difference just perceptible with certainty increases continuously as N'' is increased. Nevertheless, it has seemed unnecessary and impracticable to attempt any finer gradation of the tolerances than those just given.

These tolerances were based upon the writer's chromaticity discrimination at various points on the scale of 35 yellow plus N'' red ($35Y+N''R$). A determination of this sort was made in August 1927 at 35Y plus 7.6R (an important point on the scale in the oil industry) [18] before the duties of routine testing of Lovibond red glasses were assigned. A difference of 0.16 [19] of an N'' unit was perceptible with certainty and a difference of 0.093 of an N'' unit was named correctly eight out of ten times. A few months later, a difference of N'' equal to 0.10 at $35Y+7.6R$ was perceived with certainty. The following table shows the approximate minimum differences perceptible [20] with certainty by the writer near the points indicated on the $35Y+N''R$ scale:

Point on scale ($35Y+N''$ R)	$N''=7.6$	10.0	17.1	19.7
Minimum difference in N'' perceptible with certainty	$=0.10$	.10	.20	.30

It is seen that these differences do not exceed the tolerances; that is, in no case would a difference as large as the tolerance go undetected even in a single observation.

[18] "Prime Summer Yellow Cotton Seed Oil must be * * * of no deeper color than the two combined standard glasses of 35 yellow and 7.6 red on Lovibond's equivalent color scale * * *" Rule 61 of the rules governing transactions between members of the National Cottonseed Products Association, Inc. It is interesting to note that the occasion for the introduction of Lovibond glasses into the United States, which occurred about 1900, was the establishment of this standard of color for Prime Summer Yellow Cotton Seed Oil.

[19] Tests of Color Sense of A.O.C.S. Members and Data on Sensibility to Change in Lovibond Red, by I. G. Priest, Oil and Fat Industries, 5, pp. 66–67, March 1928.

[20] An analysis of extensive data on 36 Lovibond red glasses submitted for test in terms of our standard glasses, showed that the precision and reproducibility to be expected from this chromaticity discrimination were being realized (D. B. Judd, Chromaticity Sensibility to Stimulus Differences, Jour. Opt. Soc. Am., 22, p. 98, February 1932). It also showed that the routine procedure involves enough determinations to keep the error well within the tolerances.

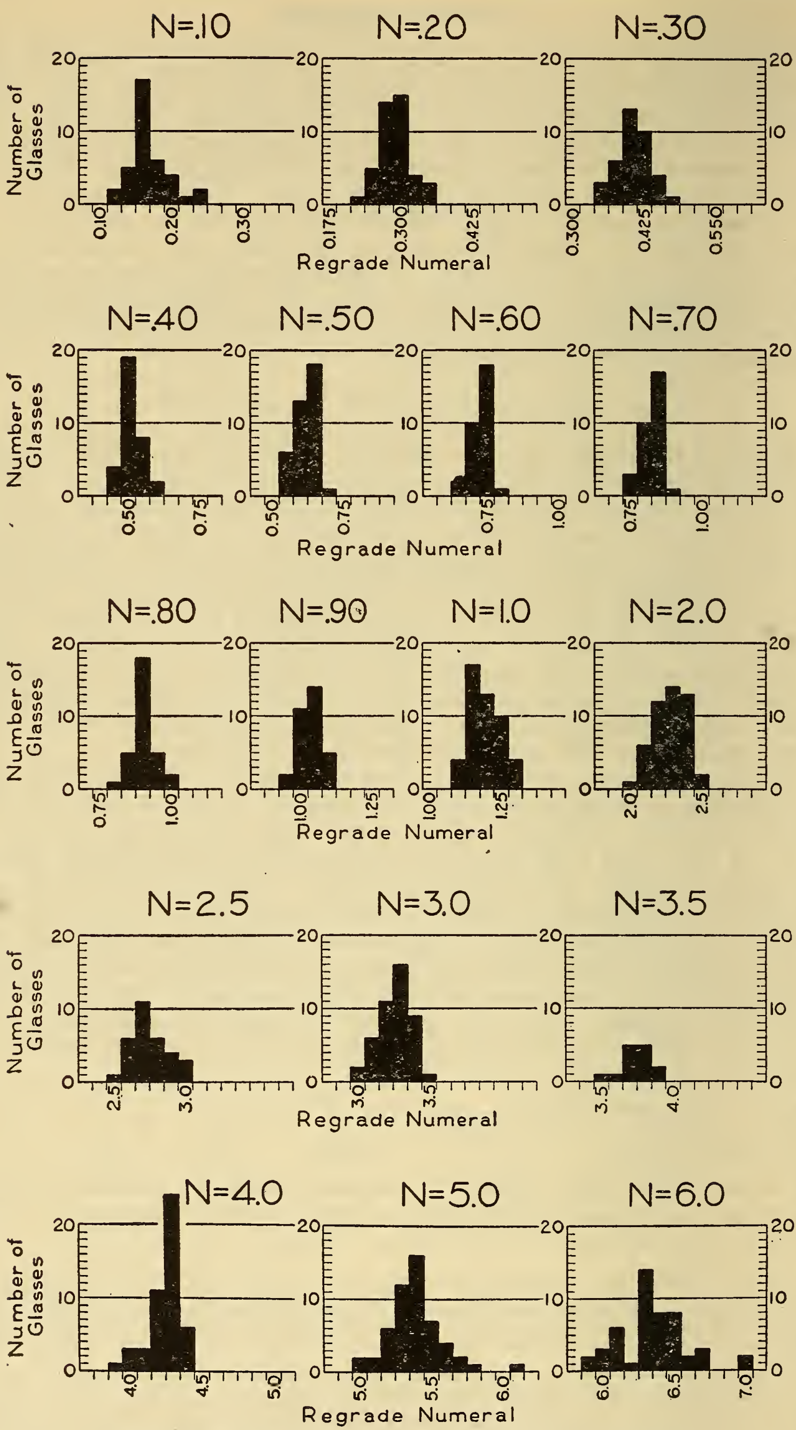

FIGURE 2.—*Frequency distributions of regrade numerals.*

The reliability of the regrade numerals is further emphasized from the evidence of uniformity obtained in the intercomparison or cross-checking of the glasses tested.

VI. DISCUSSION OF RESULTS

1. VARIATIONS AMONG GLASSES OF IDENTICAL NOMINAL GRADE

The results of this phase of the statistical investigation are presented both in graphical and tabular form. The variations in the

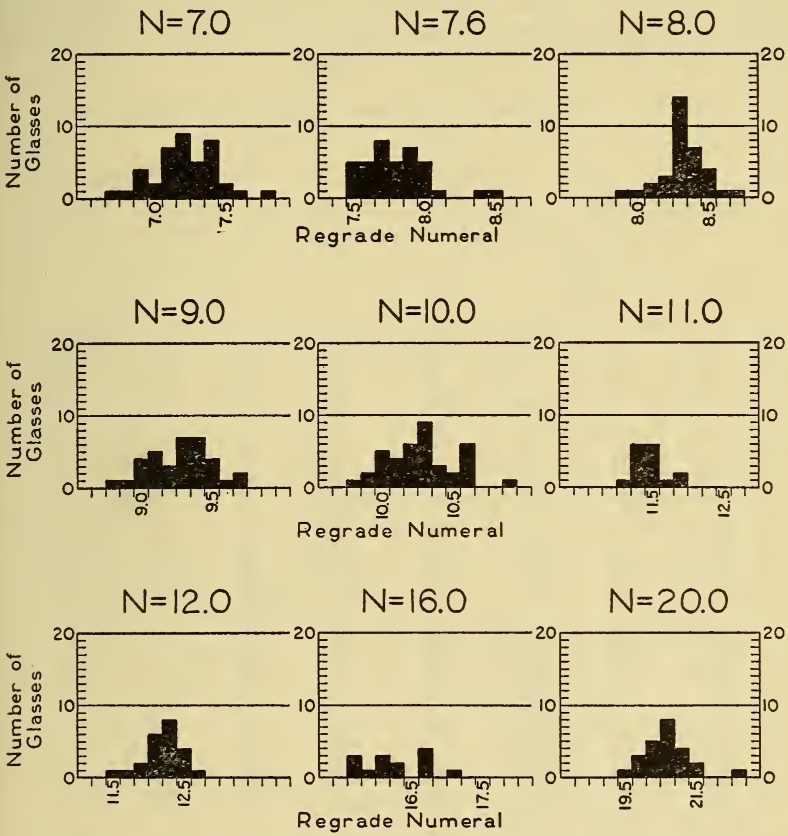

FIGURE 3.—*Frequency distributions of regrade numerals (continued).*

Figures 2 and 3 show the distributions of regrade numerals, according to the Priest-Gibson (N'') scale, for each group of Lovibond red glasses having nominal grade, N. The ordinate indicates the number of glasses having the regrade numeral shown on the abscissa. Each group is composed of glasses having identical maker's numerals engraved upon them, and no group shown contains fewer than 14 glasses. The regrade numerals, N'', are certified to be correct within certain tolerances depending upon the nominal grades of the glasses. The following table shows the tolerances which have been assigned to the ranges of nominal grades indicated:

Range	Tolerance
$0.01 \leq N \leq 10.0$	$N''=0.1$
$10.0 < N \leq 16.0$	$N''=.2$
$16.0 < N \leq 20.0$	$N''=.4$

regrade numerals, N'', occurring within the larger groups of glasses (912 glasses in all) of identical grade, N, are illustrated in figures 2 and 3. These column diagrams represent the frequency distributions of the regrade numerals for glasses of identical nominal grade.

The means of the regrade numerals of glasses composing each group are given in table 2, column 3, and are plotted as circles in figure 4, the vertical lines indicating the ranges between the minimum and maximum values of the regrade numerals in each group. These minimum and maximum values and the ranges of the regrade numerals are also given in table 2, columns 4, 5, and 6. The ranges of the regrade numerals vary from 0.12 to 3.23 N'' units, depending upon the nominal grades of the groups; in every case these ranges of N'' exceed the tolerances specified in section V, 3, in many cases by a factor of five or more. It may be noted that the larger groups with nominal grades from $N=5.0$ to $N=12.0$ have on the average a range of regrade numerals as great as unity.

TABLE 2.—*Variations among glasses of identical nominal grade*

Nominal Grade N	Number of glasses	Regrade numerals, N''				Deviations from mean in column 3	
		Mean	Minimum	Maximum	Range	Average	Maximum
1	2	3	4	5	6	7	8
0.10	37	0.16	0.11	0.24	0.13	0.02	0.08
.20	42	.29	.22	.36	.14	.02	.07
.30	37	.41	.34	.47	.13	.02	.07
.40	33	.51	.45	.59	.14	.03	.08
.50	38	.62	.55	.68	.13	.03	.07
.60	31	.73	.66	.78	.12	.02	.07
.70	31	.83	.76	.88	.12	.03	.07
.80	31	.90	.82	1.00	.18	.03	.10
.90	32	1.04	.96	1.12	.16	.03	.08
1.0	48	1.20	1.11	1.30	.19	.04	.10
2.0	48	2.27	2.02	2.47	.45	.09	.25
2.5	31	2.74	2.54	2.97	.43	.09	.22
3.0	45	3.25	2.97	3.47	.50	.10	.28
3.5	14	3.73	3.51	3.89	.38	.08	.22
4.0	48	4.25	3.93	4.43	.50	.08	.32
5.0	53	5.38	4.99	6.07	1.08	.14	.69
6.0	49	6.36	5.93	6.98	1.05	.18	.62
7.0	41	7.21	6.74	7.83	1.09	.17	.62
7.6	38	7.80	7.46	8.52	1.06	.18	.72
8.0	34	8.32	7.91	8.70	.79	.12	.41
9.0	35	9.27	8.80	9.70	.90	.17	.47
10.0	39	10.27	9.77	10.90	1.13	.19	.63
11.0	16	11.47	11.10	11.92	.82	.18	.45
12.0	23	12.22	11.53	12.61	1.08	.20	.69
16.0	14	16.26	15.61	17.06	1.45	.38	.80
20.0	24	20.69	19.51	22.74	3.23	.50	2.05

As a further indication of the variations or dispersions of glasses of identical grades, the mean and maximum deviations from the mean regrade numerals are given in table 2, columns 7 and 8. Approximately 25 percent of the total number of glasses have regrade numerals which differ from the respective means of the groups by more than our tolerances.

Not only is the range within any group large, but in most cases an overlapping of regrade numerals exists in two adjacent groups (fig. 4). In some instances, especially among the larger groups from $N=5.0$ to $N=12.0$, glasses of similar chromaticity have nominal grades differing by a nominal unit.

2. FAILURE OF ADDITIVITY IN THE LOVIBOND RED SCALE WHEN COMBINED WITH 35Y

Because of the size of the variations just shown among glasses of the same nominal grade, it is not surprising that the glasses in any

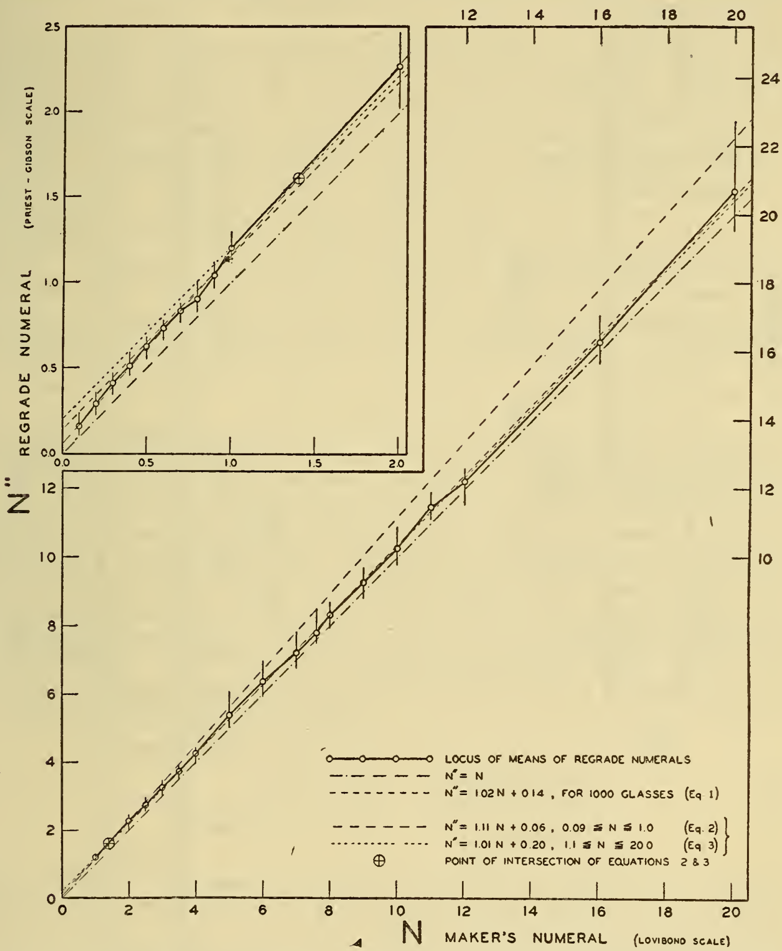

FIGURE 4.—*Graphical summary of results showing the variations among glasses of the same nominal grade (N) and the nature of the disagreement between the N'' and N scales using data on the glasses graded.*

The inset shows the data for N less than 2.0. The vertical lines indicate the ranges between the minimum and maximum values of the regrade numerals in each group. It is seen that the plotted points representing the average N'' values of the groups do not fall along the line $N''=N$. No simple adjustment can bring the N'' and N scales into agreement for the general disagreement is complex, being both erratic as well as regular in nature. The latter permits the data to be satisfactorily represented by two straight lines (equations 2 and 3), one for low and one for high values of N. However, it may be seen that the variations of regrade numerals (N'') among glasses of the same nominal grade (N) constitute the major source of error.

one set fail to satisfy the additivity condition. This failure of additivity, due to the erratic variations in the size of intervals between successive Lovibond tenth and unit glasses, is well illustrated by a series of 29 glasses selected from the Bureau's set, B.S. 9940, which

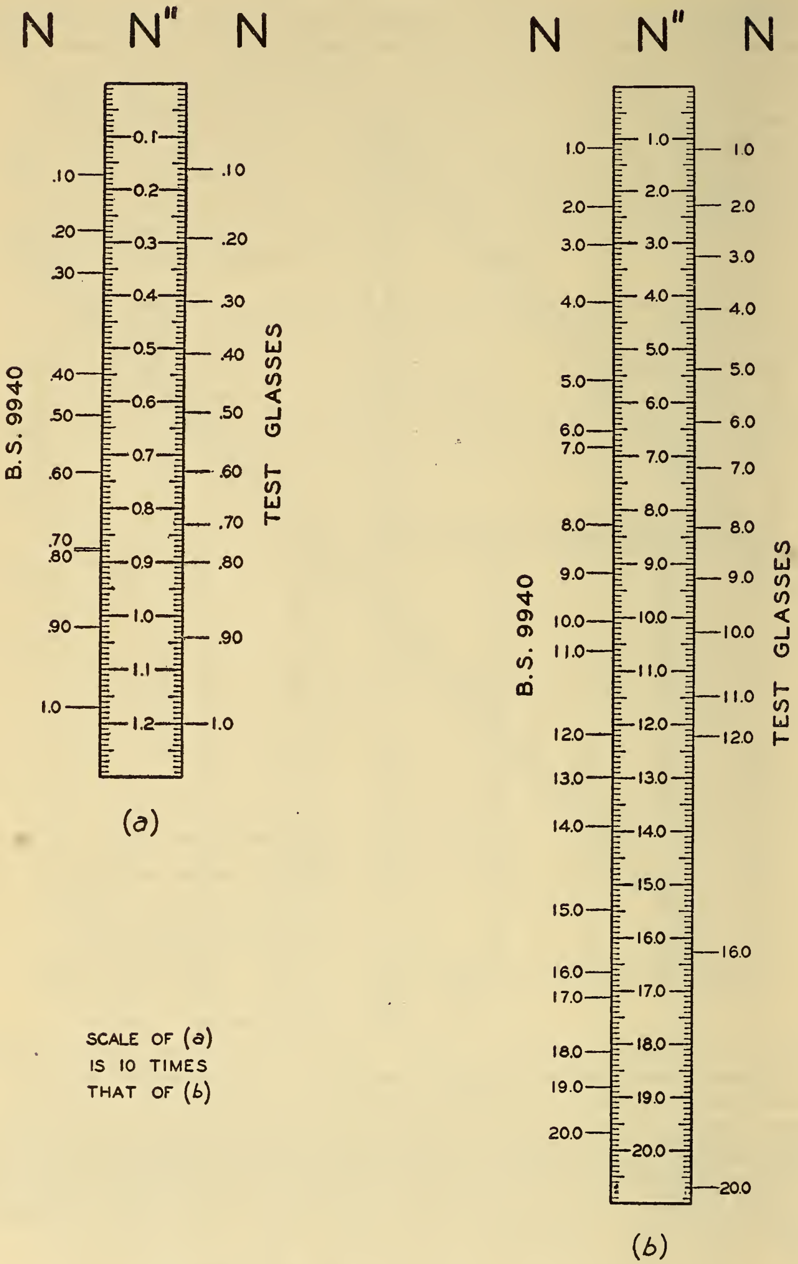

FIGURE 5.—*Erratic nature of the Lovibond scale.*

The variations occurring among glasses of an individual set, B.S. 9940, may be compared with the average variations found among the glasses sent in for calibration.

range in N from 0.10 to 1.0 in multiples of 0.1 (tenth glasses) and from 1.0 to 20.0 in multiples of 1.0 (unit glasses). Figure 5 shows these nonuniform N intervals in terms of the standard scale, N''.

It might be expected that if sufficient glasses of each nominal grade were calibrated, the erratic variations in their intervals would average out, so that on the average the scale would be additive. However, this was not the case with the glasses studied and reported herein, as may be seen in figure 4 and more clearly in figure 5, where the mean regrade numerals, given in column 3 of table 2, are also marked off on the standard scale. The nominal tenth intervals vary from 0.07 to 0.16 N'' units and the nominal unit intervals from 0.85 to 1.13 N'' units. Hence, this particular group of nominal tenth and unit glasses (829 in number) fails to follow an additive scale and it is unlikely that a still larger group would do so.

Moreover, this failure to follow an additive scale is not entirely erratic. It may be analyzed into regular as well as erratic deviations from the Priest-Gibson scale, the additivity of which has been confirmed. A straight-line equation in N'' and N was found by least-squares adjustment of the data for the 1,000 glasses. This equation is:

$$N'' = 1.02\,N + 0.14 \tag{1}$$

The slope of this line differs from unity, but since any scale related to the N'' scale by any constant factor is additive, this deviation from unity is of no consequence in this connection. However, the existence of the significant intercept is evidence that the N scale deviates in a regular way from conformity to the additivity condition.

A more accurate description of this regular departure from additivity may be obtained by using two straight lines instead of one. The plotted points shown in figure 4 suggest one straight line extending from $N=0.09$ to $N=1.0$, and another extending from $N=1.1$ to $N=20.0$. Equations for two such lines were found by least-squares adjustment. The equations are:

$$N'' = 1.11\,N + 0.06, \quad 0.09 \leqq N \leqq 1.0 \tag{2}$$

$$N'' = 1.01\,N + 0.20, \quad 1.1 \leqq N \leqq 20.0 \tag{3}$$

The straight lines corresponding to all three equations are shown on figure 4. It will be noted that equations (2) and (3), which agree at the point ($N=1.4$, $N''=1.614$), fit the plotted values considerably better than equation (1); hence they describe more accurately the relation between the N and N'' scales. The intercepts are again evidence of a regular departure from additivity even for the lower section of the scale. Furthermore, the greater slope of equation (2) indicates that the size of the N unit is larger for the lower section of the scale ($N<1.4$) than for the upper ($N>1.4$). If the glasses of the lower section are used in combination with those of the upper, this disparity in the size of the N unit will in itself also introduce errors of nonadditivity. This is a second regular type of failure of the N scale to satisfy the additivity condition, and is distinct from that indicated by the intercepts.

A still more accurate description of the variations in the size of the N unit might possibly be accomplished by setting up equations of the second or higher degree. However, such refinement is not

required here, since the object has been to show the degree to which the Lovibond scale, embodied in these 1,000 red glasses combined with 35Y, fails to be additive. Both regular and erratic departures from additivity have been demonstrated. Of the two, the erratic departures are the more important; it has been shown that the discrepancies between glasses of the same nominal grade frequently amount to a whole unit.

WASHINGTON, October 13, 1933.